# SARX Model Application for Industrial Power Demand Forecasting in Brazil

# Aplicação do Modelo SARX para Previsão do Consumo Industrial de Energia Elétrica no Brasil

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## Abstract

The objective of this paper is to propose the application of the SARX model to arrive at industrial power consumption forecasts in Brazil, which are critical to support decision-making in the energy sector, based on technical, economic and environmentally sustainable grounds. The proposed model has a seasonal component and considers the influence of exogenous variables on the projection of the dependent variable and utilizes an autoregressive process for residual modeling so as to improve its explanatory power. Five exogenous variables were included: industrial capacity utilization, industrial electricity tariff, industrial real revenues, exchange rate, and machinery and equipment inflation. In addition, the model assumed that power forecast was dependent on its own time lags and also on a dummy variable to reflect 2009 economic crisis. The study used 84 monthly observations, from January 2003 to December 2009. The backward method was used to select exogenous variables, assuming a 0.10 descriptive value. The results showed an adjusted coefficient of determination of 93.9% and all the estimated coefficients were statistically significant at a 0.10 descriptive level. Forecasts were also made from January to May 2010 at a 95% confidence interval, which included actual consumption values for this period. The SARX model has demonstrated an excellent performance for industrial power consumption forecasting in Brazil.
**Keywords**: Time Series. Energy. SARMAX.

## Resumo

O objetivo deste trabalho é propor a aplicação do modelo SARX para projetar o consumo industrial de energia elétrica no Brasil, fundamental para o planejamento de capacidade do setor de energia do país. O modelo proposto inclui componente sazonal e considera a influência de variáveis exógenas e modela os resíduos por meio de um processo auto-regressivo a fim de aumentar o poder explicativo do modelo. Variáveis exógenas consideradas foram utilização da capacidade da indústria; tarifa de energia elétrica; faturamento real da

Artigo publicado anteriormente nos Anais do TACTiCS Global Conference em 2011.
Artigo submetido em 17 de novembro de 2011 e aceito em 30 de março de 2012 pelo Editor Marcelo Alvaro da Silva Macedo, após *double blind review*.

indústria; taxa de câmbio; índice de inflação de máquinas e equipamentos. O modelo também incluiu uma variável dummy para refletir a crise econômica de 2009 e o próprio consumo de energia elétrica, em tempos defasados. O estudo abrangeu 84 observações mensais, de janeiro de 2003 a dezembro de 2009. Aplicou-se o método *backward* para a seleção de variáveis exógenas, considerando um nível descritivo de 0,10. O modelo apresentou um coeficiente de determinação ajustado de 93,9% e todos os coeficientes estimados foram estatisticamente significantes a um nível descritivo de 0,10. Os valores projetados de janeiro a maio de 2010 foram incluídos no intervalo de confiança a 95%. O modelo SARX apresentou uma ótima acuracidade na projeção do consumo de eletricidade no setor industrial do Brasil.
**Palavras-chave**: Séries Temporais. Energia. SARMAX.

## 1. Introduction

Decision making in the energy sector and its effect on the development of Brazilian infrastructure relies heavily on accurate forecasts of demand. Forecasts on different time horizons are vital tools for handling the complexity of the national energy system and the day-to-day operations of the country's power plants.

The Ministry of Mines and Energy is responsible for the conception, articulation and coordination of Brazil's energy planning and it releases the 10-Year Energy Plan, which is prepared by the Energy Research Agency (EPE) and considers short, medium and long-term outlooks. According to the EPE (2009), the 10-Year Energy Expansion Plan for 2008-2017 contains important signals to guide actions and decisions aimed at evening the balance between the country's economic growth forecasts, their impacts on energy requirements and the necessary expansion of supply, based on technical, social, economic and environmental dimensions.

Furthermore, according to Miranda (2009), the very short term forecast, which involves high frequency data, is also essential for the reliability and efficient operation of the power sector, allowing the supply to be allocated efficiently, in addition to identifying potential distortions in the upcoming periods (days, hours or fractions of hours).

When it comes to generating power, Brazil stands out on the global stage as hydroelectric power represents 79.6% of its total installed capacity, while 12.8% of the total capacity comes from fossil fuels, according to EPE' 10-Year Energy Plan for 2008-2017 (2009). In contrast, reveals Goldenberg (2004), approximately 80% of all the energy consumed around the world comes from fossil fuels and petroleum derivatives.

A quality forecast of future electricity demand, therefore, is essential to streamline the management of the energy system and its operational processes.

Several different models have been used to forecast electricity consumption, such as the Box-Jenkins model, cointegration and distributed lag models, and structural time series models.

Earlier studies have been conducted on forecasting electricity consumption in Brazil using econometric methodologies. The study by Modiano (1984) was one of the first to measure the income and price elasticities of electricity in Brazil based on annual data from 1963 to 1981. Modiano (1984) used the method of least squares with correction for serial correlation by the Corchranne-Orcutt technique. The results showed that only the long-term income elasticities were elastic in relation to electricity consumption.

Andrade and Lobão (1997) made an analysis of residential electricity consumption in Brazil for the period from 1963 to 1995, by estimating the income and price elasticities of aggregate demand. This study is, to some extent, an update of the work by Modiano (1984), and it demonstrated that the short-term income and price elasticities were higher than the long-term elasticities.

Schmidt and Lima (2004) estimated long-term price and income elasticities by means of cointegration models for the residential, industrial and commercial consumption categories. The estimated income elasticity obtained was higher than one, while the price elasticity, in modulus, was lower than one. They also made electricity demand predictions for 2001 to 2005.

Irffi *et al*. (2009) estimated electricity demand for the residential, commercial and industrial categories in the Northeast region of Brazil, in the period from 1970 to 2003, using the Dynamic

Ordinary Least Squares (DOLS) method, and they made predictions for the period from 2004 to 2010. The results obtained matched the conclusions of the previous studies in the Brazilian literature for the residential, industrial and commercial categories.

Studies conducted in other countries on forecasting electricity consumption have involved such models as unit root test, DOLS, vector autoregressive (VAR) and vector error correction (VEC). Donatos and Mergos (1991) estimated residential electricity demand in Greece, both for the short and long-term, based on data from 1961 and 1986.

In the United States, Silk e Joutz (1997) used the VAR/VEC model to estimate residential power demand, based on annual residential electricity data from 1949 to 1993, and they made forecasts for 1994 and 1995.

Zachariadis and Pashourtidou (2007), in Cyprus, used the VEC and unit root test models to forecast electricity consumption in the residential and services sectors. The results indicated that the services sector is less elastic and reverts faster to equilibrium than the residential sector.

The objective of this paper is to apply the SARX model to forecast industrial electricity consumption in Brazil. The SARX model has a seasonal component and considers several exogenous economic variables in its electricity consumption forecasts and, additionally, it uses an autoregressive process for residual modeling so as to improve the explanatory power of the model. The study included 84 monthly observations, from January 2003 to December 2009.

## 2. Box-Jenkins Approach

According to Hahn (2002), the time series models of Box and Jenkins (1970) are among the oldest for making forecasts of power demand. One simple forecasting approach consists of the ARMA(p,q) models, applicable for stationary series. A process $\{X_t, t = 0, t = \pm 1, \pm 2, \dots\}$ is considered an ARMA(p,q) process if $\{X_t\}$ is stationary and if, for each t (Brockwell & Davis, 1991), the equation below is observed:

$$X_t - \emptyset_1 X_{t-1} - \dots - \emptyset_p X_{t-p} = Z_t + \theta_1 Z_{t-1} + \dots + \theta_q Z_{t-q} \quad (1)$$

where the process $\{Z_t\}$ is a white noise with mean zero and constant variance with normal distribution and independent realizations, i.e., $\{Z_t\} \sim N(0, \sigma^2)$. Using the lag operator B, where $B^s X_t = X_{t-s}$, and the operators $\emptyset(B) = 1 - \emptyset_1 B^1 - \dots - \emptyset_p B^p$ e $\theta(B) = 1 + \theta_1 B^1 + \dots + \theta_q B^q$, the equation (1) can be written more compactly as $\emptyset(B) X_t = \theta(B) Z_t$.

The process $\{X_t\}$ is said to be an ARMA process with mean $\mu$ if $\{X_t - \mu\}$ is an ARMA(p,q) process. The ARMA(p,q) model can be extended to the class of ARIMA models, according to which differencing is used to convert a non- stationary series into a stationary series. In the ARIMA(p,d,q) model, the dth difference of the series $\{X_t\}$ is represented by a stationary and invertible ARMA process. The general form of the ARIMA(p,d,q) model is expressed as:

$Y_t - \emptyset_1 Y_{t-1} - \ldots - \emptyset_p Y_{t-p} = Z_t + \theta_1 Z_{t-1} + \ldots + \theta_q Z_{t-q}$

where $Y_t = (1 - B)^d X_t$ , or, more compactly, as:

$\emptyset(B)(1 - B)^d X_t = \theta(B) Z_t$

The ARMA and ARIMA models are univariate and normally used to make short-term forecasts. They capture autocorrelations between the values of the series in successive points in time but do not model seasonal effects. A seasonal series with period s presents repeated observations at intervals lagged by s periods and, as a result, the series can present autocorrelations for a seasonal period s. The ARMA/ARIMA models may be generalized to take into account the effect of seasonality, obtaining the SARIMA class of models, which contain a non-seasonal component ARIMA(p,d,q) and a seasonal component with $(P, D, Q)_s$ parameters.

According to Box and Jenkins (2008), the operator $B^s X_t = X_{t-s}$ plays an important role in the analysis of seasonal series and, as non-stationarity occurs in the series $X, X_{t-s}, X_{t-2s}, \ldots,$ the simplicator operation $(1 - B^s) X_t$ is also vital in the analysis. The general form of the multiplicative model with SARIMA seasonality (p,d,q) x (P,D,Q)s may be written as (BOX; JENKINS, 2008) the equation below:

$$(1 - \emptyset_1 B^1 - \ldots - \emptyset_p B^p)(1 - \Phi_1 B^s - \ldots - \Phi_P B^{sP})(1 - B^d)(1 - B^S)^D X_t = (1 + \theta_1 B^1 + \ldots + \theta_q B^q)(1 + \Theta_1 B^s + \ldots + \Theta_Q B^{sQ})$$

## 3. ARX Model

According to Franses (1991), the ARMAX model is an extension of the ARIMA modeling and includes exogenous variables, in addition to autoregressive components and moving averages. Similarly, the random errors are independent, identically distributed and they represent a white noise with mean zero and constant variance, i.e., $\{Z_t\} \sim N(0, \sigma^2)$.

The multivariate ARMAX(p,q) model can be written by the general equation below (STOFFER; SCHUMAY, 2006):

$$\mathbf{X}_t = \Gamma \mathbf{U}_t + \sum_{j=1}^{p} \emptyset_j \mathbf{X}_{t-j} + \sum_{k=1}^{q} \theta_k \mathbf{Z}_{t-k} + \mathbf{Z}_t$$

where $X_t$ is a k x 1 vector composed of k dependent variables; $\Gamma$ is a k x r matrix of the coefficients of the exogenous variables; $U_t$ is a r x 1 vector of r exogenous variables, which may include their lags in time; $\emptyset_j$ is a k x k matrix of the coefficients of the autoregressive components, for j = 1, ...,p; $X_{t-j}$ is a k x 1 matrix of the autoregressive components of $X_t$, for j = 1, ...,p; $\theta_k$ is a k x k matrix of the coefficients of the white noise $Z_{t-k}$ of the past periods, for k = 1, ...,q; $Z_t$ is a k x 1 matrix of the white noise of the current period.

One specific case of the ARMAX(p,q) model, for q = 0, is the ARX(p) model that explains a dependent variable by means: (i) of the different exogenous variables of the current period and for past periods; (ii) of the past values themselves of the dependent variable.

The general multivariate ARX model is expressed as:

$$\mathbf{X}_t = \Gamma \mathbf{U}_t + \sum_{j=1}^{p} \emptyset_j \mathbf{X}_{t-j} + \mathbf{Z}_t$$

A variant of the ARX is the SARX model, which includes a seasonal autoregressive component $\sum_{j=1}^{P} \Phi_j X_{t-j}$ of order P and seasonal period s. The general multivariate SARX model is expressed as:

$$X_t = \Gamma U_t + \sum_{j=1}^{p} \emptyset_j X_{t-j} + \sum_{k=1}^{P} \Phi_k X_{t-k} + Z_t,$$

where the definitions expressed in equation (1) are applied. Additionally, $\Phi_k$ is a k x k matrix of the coefficients of the seasonal autoregressive components of order P and seasonal period s, for j = 1, ..., P; $X_{t-k}$ is a k x 1 matrix of the seasonal autoregressive components of $X_t$, for j = 1, ..., P.

For the univariate case $X_t$, the SARX model will be:

$$X_t = \Gamma U_t + \sum_{j=1}^{p} \emptyset_j X_{t-j} + \sum_{k=1}^{P} \Phi_k X_{t-k} + Z_t$$

Using compact notation with operators, the SARX model may be expressed as:

$$X_t = \Gamma U_t + (\emptyset_1 B^1 + \ldots + \emptyset_p B^P) X_t + (\Phi_1 B^S + \ldots + \emptyset_P B^{SP}) X_t + Z_t$$

where:

$X_t$: dependent variable in time t;

$\Gamma = (\Gamma_1, \ldots, \Gamma_r)$: vector of coefficients of the exogenous variables;

$U_t = (U_1, \ldots, U_r)$: vector of exogenous variables that can include current times and past times;

$\emptyset_1 B^1 + \ldots + \emptyset_p B^P$ : autoregressive and non-seasonal part of order P;

$\Phi_1 B^1 + \ldots + \Phi_P B^{sP}$ : autoregressive and seasonal part of order P and seasonal period s;

$Z_t$: white noise such that $\{Z_t\} \sim N(0, \sigma^2)$

## 4. Results and Discussion

### 4.1. Industrial Power Consumption Series

The data group used consisted of an electricity consumption series for the industrial sector in Brazil and included 84 monthly observations, from January 2003 to December 2009, obtained on the IPEADATA website. Figure 1 shows the evolution of industrial electricity consumption in MWh from January 2000 to December 2009.

According to the 10-Year Plan for 2008-2017, prepared by the Energy Research Agency (EPE), a body of the Ministry of Mines and Energy, electricity consumption is calculated by power subsystem (there being four interconnected subsystems – North, Northeast, Southeast/Center-West and South, as well as isolated systems that are not connected to the national power grid) and also by consumption category, namely residential, industrial, commercial and others (government, street lighting, public services and own consumption).

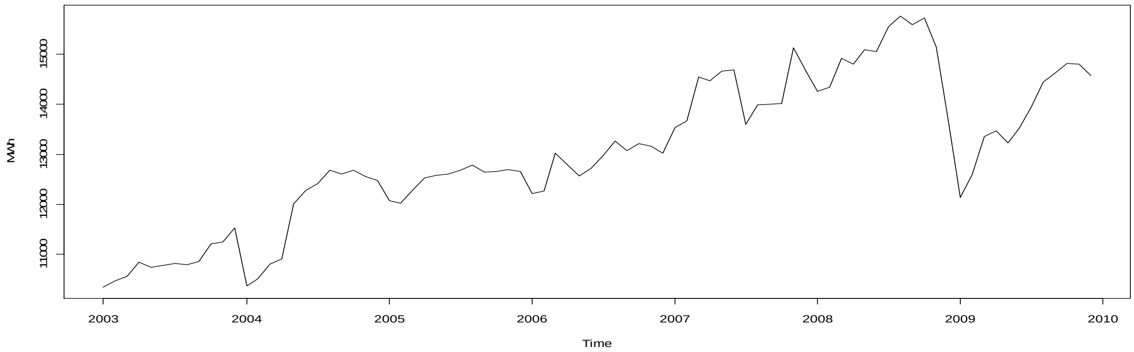



**Figure 1 - Industrial Power Consumption (GWh)**

In October 2008, the growth in the consumption of electricity by Brazilian industry in the deseasonalized series (considering a moving average of 12 months ending in the reference month) was the lowest since April 2007. Consumption had been declining since January 2007, after a long period of expansion that lasted from May 2006 to December 2007.

This decline in 2008 can be explained by the changes in the international economic climate in 2008, which affected the evolution of industrial electricity consumption in Brazil. Furthermore, according to the EPE, the seasonality effect needs to be taken into consideration: after strong growth rates, such as those reported in 2007, lower subsequent rates were to be expected.

The consumption of 15,148 GWh registered in November 2008 is almost the same as the figure in November 2007, of 15,135 GWh. In December 2008, the consumption of electricity by industry, of 13,698 GWh, was 6.7% lower than in December 2007, and it was also the lowest monthly figure since August 2008.

Across the country, the only region to report growth in 2008 over 2007 was the Center-West. In the Southeast, where 57% of industrial power consumption is concentrated, the reduction reached 10.1%. In São Paulo, many factories gave their workers mandatory vacations and brought forward planned maintenance work, particularly automobile manufacturers and auto parts suppliers. In Rio de Janeiro, metallurgy and chemicals were the most affected subsectors. Valesul cut its production of primary aluminum by 20% and 50%, respectively, in November and December. In Minas Gerais, Usiminas brought forward the maintenance work on one of its blast furnaces, and Vale cut its iron ore and manganese extraction activities and its ferroalloy production.

Power consumption by industry registered a sharp decline in January 2009 (Figure 2) compared to January 2008, down 14.9%. Consumption in January 2009 was 12,144 GWh, standing at around the same level as in January 2005, which was 12,078 GWh. This retraction in the industrial market reflected the intensification of the effects of the international financial crisis on the Brazilian economy, with export-intensive sectors taking the hardest hit.

The recovery of industrial consumption began in February 2009, albeit sluggishly. Following a stronger increase in the first quarter of the year, consumption fell back in April. In May 2009, industries demanded 13,531 GWh from the national power grid, representing a drop of 12.4% from May 2008. The recovery in industrial consumption in 2009 occurred across all regions, although more intensely in the Southeast, where the effects of the financial crisis were greater.

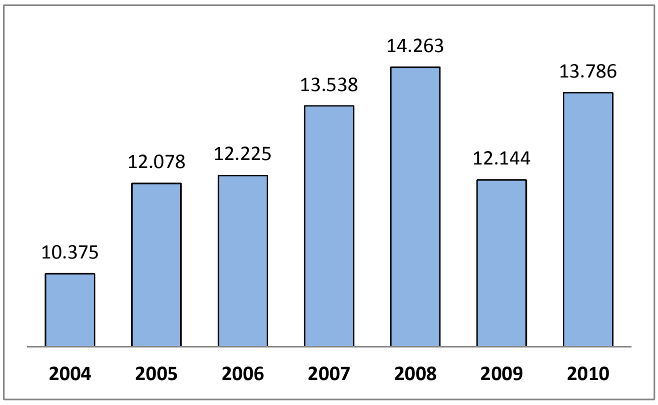



**Figure 2 – Industrial electricity consumption in GWh (months of January, 2004-2010)**

The recovery of industrial consumption began in February 2009, albeit sluggishly. Following a stronger increase in the first quarter of the year, consumption fell back in April. In May 2009, industries demanded 13,531 GWh from the national power grid, representing a drop of 12.4% from May 2008. The recovery in industrial consumption in 2009 occurred across all regions, although more intensely in the Southeast, where the effects of the financial crisis were greater.

Brazilian industrial consumption totaled 165,536 GWh in 2009, or 8% lower than in 2008. This consumption amount stood between the figures for 2006 (154,315 GWh) and 2007 (171,013 GWh). It is important to observe, however, that industrial consumption did make a slow recovery throughout 2009, as shown in the quarterly percentage changes compared to the same periods the year before: -12.5% in the first quarter, -10.5% in the second, -8.3% in the third and -0.8% in the fourth.

Due to the economic recovery experienced by Brazilian industry and, given the effects of the international financial crisis, the low comparative figures in 2009, industrial power consumption has presented high rates of growth in 2010. In May 2010, consumption rose 16.6% from the same month in 2009, and in the five months to May, it increased 14.2% from a year before. The increase in power consumption has reflected the growth in Brazil's industrial output, which was up 18% in April 2010. According to the IBGE statistics institute, the industry has been expanding across the board: of the 27 industrial activities surveyed by the institute, only two did not report any growth in the first four months of 2010 compared to the same period in 2009. Having suffered the most from the financial crisis, the iron ore extraction and metallurgy sectors reported a particularly strong recovery, up 50.7% and 34.1%, respectively.

## 4.2.SARX Model Results

Industrial electricity is influenced by economy growth and industry output level. This influence was represented in this work by five exogenous variables, namely industrial capacity utilization, industrial electricity tariff at December 2009 prices, industry revenues, exchange rate, and machinery and equipment inflation. A dummy variable was included to reflect 2009 global economic crisis, assuming values equal to 1, for all months of 2009 and zero in the remaining monthly periods. Moreover, power consumption was assumed to be dependent on its own past time lags. The SARX model was adjusted in EViews software (version 7).

Figure 3 shows the autocorrelation (ACF) and partial autocorrelation (PACF) functions of the industrial power consumption series. ACF indicates a gradual decay, with positive correlations and out of the confidence interval up to lag 19. Correlations are positive until lag 30th and negative from lags 31to 40. PACF shows statistically significant values only for lags 1 and 13.

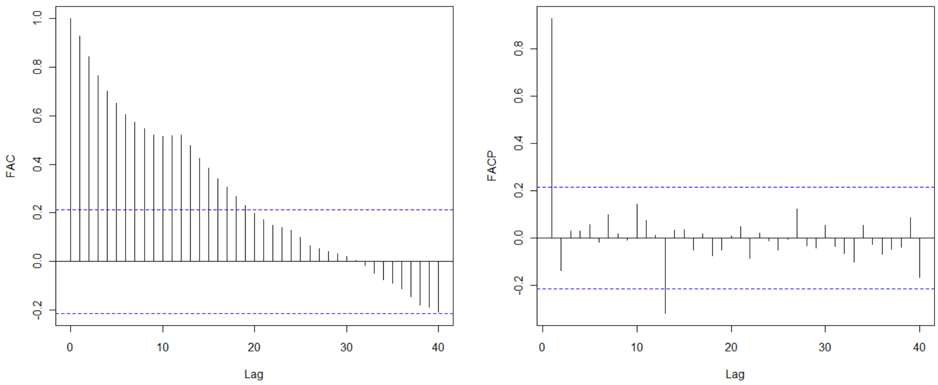
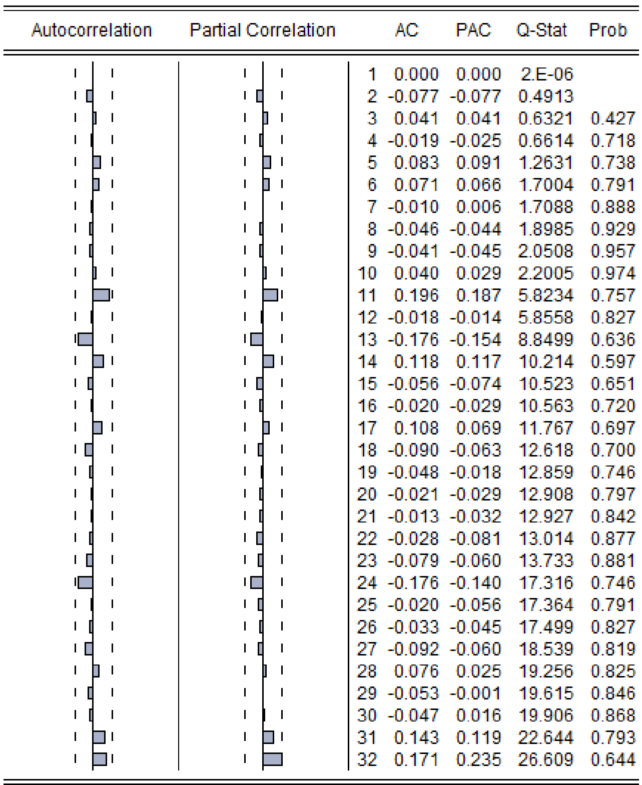



**Figure 3 – ACF and PACF of the industrial power consumption series**

Initially, to prepare the multiple linear regression model, the aforementioned exogenous variables were used at time t and their lags at five times (t-1, t-2, t-3, t-4, t-5). The backward method was applied to select exogenous variables, considering a 0.10 descriptive level for the exclusion of the variables.

The residual analysis of the multiple linear regression indicated in Figure 4 demonstrates that the auto-correlation and partial auto-correlation functions do not show statistically significant values different from zero, suggesting a white noise behavior and appropriate model adjustment.



| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.000 | 0.000 | 2.E-06 | |
| 2 | -0.077 | -0.077 | 0.4913 | |
| 3 | 0.041 | 0.041 | 0.6321 | 0.427 |
| 4 | -0.019 | -0.025 | 0.6614 | 0.718 |
| 5 | 0.083 | 0.091 | 1.2631 | 0.738 |
| 6 | 0.071 | 0.066 | 1.7004 | 0.791 |
| 7 | -0.010 | 0.006 | 1.7088 | 0.888 |
| 8 | -0.046 | -0.044 | 1.8985 | 0.929 |
| 9 | -0.041 | -0.045 | 2.0508 | 0.957 |
| 10 | 0.040 | 0.029 | 2.2005 | 0.974 |
| 11 | 0.196 | 0.187 | 5.8234 | 0.757 |
| 12 | -0.018 | -0.014 | 5.8558 | 0.827 |
| 13 | -0.176 | -0.154 | 8.8499 | 0.636 |
| 14 | 0.118 | 0.117 | 10.214 | 0.597 |
| 15 | -0.056 | -0.074 | 10.523 | 0.651 |
| 16 | -0.020 | -0.029 | 10.563 | 0.720 |
| 17 | 0.108 | 0.069 | 11.767 | 0.697 |
| 18 | -0.090 | -0.063 | 12.618 | 0.700 |
| 19 | -0.048 | -0.018 | 12.859 | 0.746 |
| 20 | -0.021 | -0.029 | 12.908 | 0.797 |
| 21 | -0.013 | -0.032 | 12.927 | 0.842 |
| 22 | -0.028 | -0.081 | 13.014 | 0.877 |
| 23 | -0.079 | -0.060 | 13.733 | 0.881 |
| 24 | -0.176 | -0.140 | 17.316 | 0.746 |
| 25 | -0.020 | -0.056 | 17.364 | 0.791 |
| 26 | -0.033 | -0.045 | 17.499 | 0.827 |
| 27 | -0.092 | -0.060 | 18.539 | 0.819 |
| 28 | 0.076 | 0.025 | 19.256 | 0.825 |
| 29 | -0.053 | -0.001 | 19.615 | 0.846 |
| 30 | -0.047 | 0.016 | 19.906 | 0.868 |
| 31 | 0.143 | 0.119 | 22.644 | 0.793 |
| 32 | 0.171 | 0.235 | 26.609 | 0.644 |

**Figure 4 - Auto-correlation and partial auto-correlation functions for the proposed model**

The coefficients of the adjusted model are presented in Table 2, where:
Dummy: variable to reflect economic crisis in 2009, assuming values equal to 1 in all months of 2009 and zero in all remaining monthly periods;
Fat: real industry revenue index in Brazil (Base 100 = December 2009);
Cap: Industry capacity utilization in percentage;
Camb: Exchange rate in terms of local currency (Brazilian Real) per US dollars (R$/US$);
Infl: inflation index for machinery and equipment as measured by Brazil's IPA-EP (index 100 = December 2009);
Tarifa: industrial power tariff in real terms (index 100 = December 2009), measured in R$/MWh;
AR(1): auto-regressive component of first order;
SAR(4): seasonal auto-regressive component with a period of 4 months.

Note that the descriptive levels associated with the estimated coefficients are all lower than 0.10. Resulting adjustment coefficient of determination was 93.9%.

| Variables | Coefficients | Standard Deviation | t-Student | Descriptive Level |
|---|---|---|---|---|
| Dummy | 814.252 | 258.374 | -3.151 | 0.002 |
| Fat | 29.788 | 9.327 | 3.194 | 0.002 |
| Cap | 84.040 | 31.301 | 2.685 | 0.009 |
| Camb | 1048.092 | 286.829 | -3.654 | 0.001 |
| Infl | 97.678 | 16.240 | 6.015 | 0.000 |
| Tarifa | 11.985 | 4.695 | -2.553 | 0.013 |
| AR(1) | 0.694 | 0.063 | 7.552 | 0.000 |
| SAR(4) | 0.227 | 0.120 | -1.892 | 0.063 |

**Table 2 – Adjusted model coefficients**

The equation of the obtained SARX model is expressed by:

$$Cons_t = -814.252.Dummy + 29.788.Fat_t + 84.040.Cap_t - 1048.092.Camb_t + 97.678.Infl_t - 11.985.Tarifa(t) + u(t),$$

where $(1- 0,6944\ B).(1- 0,2265\ B^4)\ .u(t) = \varepsilon(t)$

Figure 5 presents actual power consumption from January 2003 to December 2009 and the forecast values according to the proposed SARX model within the 95% interval confidence.

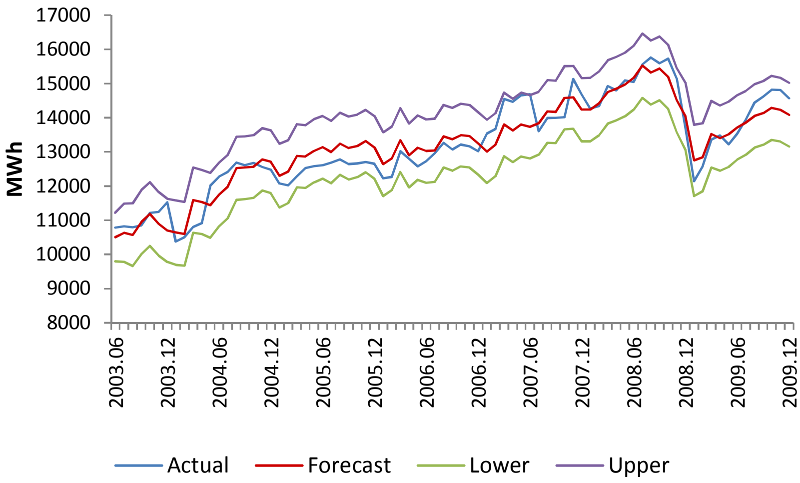



**Figure 5 – Values adjusted to the demand series with the respective intervals with 95% confidence**

In order to forecast power consumption from January to May 2010, the values of the exogenous variables presented in Table 3 were considered. Table 4 presents the projected values and the confidence intervals at 95%. Note that the actual values are included in the confidence interval at 95% from the SARX.

| Month | Dummy | Fat | Cap | Camb | Infl | Tarifa |
|---|---|---|---|---|---|---|
| 01/10 | 1 | 83.9161 | 78.6 | 1.780 | 100.380 | 224.476 |
| 02/10 | 0 | 86.8007 | 78.8 | 1.840 | 101.192 | 229.158 |
| 03/10 | 0 | 108.042 | 81.7 | 1.786 | 101.141 | 224.272 |
| 04/10 | 0 | 97.1154 | 82.3 | 1.758 | 101.337 | 219.919 |
| 05/10 | 0 | 102.360 | 82.9 | 1.813 | 102.146 | 216.055 |

**Table 3 – Values of exogeneousvariables considered for forecast from January to May 2010**

| Month | Actual (GWh) | Forecast (GWh) | Lower Limit of the CI at 95% | Lower Limit of the CI at 95% |
|---|---|---|---|---|
| 01/10 | 13.786 | 13.486 | 12.536 | 14.436 |
| 02/10 | 14.439 | 14.267 | 13.261 | 15.272 |
| 03/10 | 15.000 | 15.254 | 14.257 | 16.252 |
| 04/10 | 15.327 | 15.106 | 14.132 | 16.080 |
| 05/10 | 15.411 | 15.330 | 14.384 | 16.276 |

**Table 4 - Comparison of the forecast and actual values of electricity consumption from January to May 2010**

## 5. Conclusion

This paper, which is empirical in nature, is intended to contribute to modeling the forecast of industrial electricity consumption in Brazil by means of SARX, given the need for accurate forecasting of electricity demand to guide decision-making aimed at evening the balance between the country's economic growth forecasts, their impacts on energy requirements and the necessary expansion of supply, based on technical, economic and environmentally sustainable grounds. The study used 84 monthly observations, from January 2003 to December 2009, excluding the effects of electricity rationing that occurred in 2001.

The advantage of using SARX, which has seasonal components lies in the adjustment of a multiple linear regression model that includes relevant exogenous variables, in conjunction with the adjustment of an autoregressive model for the consumption series itself.

The model presented an important explanatory power, as seen by an adjusted coefficient of determination of 93.9%. All estimated coefficients were statistically significant at a 0.10 descriptive level. The forecast values of electricity consumption using the ARX model obtained for the months of January and February 2010 were included in the interval with a 95% confidence level.

An extension of this study could involve developing SARX models for forecasting power consumption by the commercial sector (including real retail sales as an exogenous variable) and developing forecasts for the upcoming 12 months.

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